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# Multi-objective Economic Emission Load Dispatch with Nonlinear Fuel Cost and non-inferior Emission Level Functions for a 30-bus IEEE test case system

**Abstract**-An ideal multi-objective optimization method for economic emission load dispatch (EELD) with non-linear fuel cost and emission level functions in power system operation is presented. In this paper, the problem treats economy, emission, and transmission line security as vital objectives. The load constraints and operating constraints are taken into account. Assuming goals for individual objective functions, the multi-objective problem is converted into a unique-objective optimization by the goal-attainment method, which is then taken care of by the simulated annealing (SA) technique. The solution can offer a best compromising solution in a sense close to the requirements of the system designer. Results for a 30-bus IEEE test case system have been utilized to demonstrate the applicability and authenticity of the proposed method.

## I. INTRODUCTION

Looking at the sophistication of the power utility sectors, basically for thermal power plants, the security measures for the power system as a whole are taken into account incorporating environmental effects out of generating units with economic aspects. The basic objective of economic emission and load dispatch[1-2] is to trace out the optimal power generated in fossil-based generating units by optimizing the fuel cost attaching a squared nonlinear dependence in the cost function and a non-smooth emission level function simultaneously taking into account various inequality constraints. Zahavi and Eisenberg [3] presented a method to solve economic environmental power dispatch problem without exemplifying it. Similarly In goal programming [4], approach for EELD, the transmission loss and the line security measures were not taken into account.

A better multi-objective optimization procedure based on probability security criteria to obtain a set of non-inferior solutions was presented in [5]. Nanda et al. [6] have formulated the EELD problem with line flow constraints and solved it through a classical technique, but the mathematical formulation of the security constrained problem would require a very large number of constraints to be considered. This classical technique introduced a preference index between the two objectives (economy and emission) in order to decide on an optimal solution, and this would result in complex problem formulation when the number of objective functions exceeds two. The major disadvantage of the aforesaid methods in solving the EELD problem is that they are insufficient for handling non-smooth fuel cost and emission level functions. In other words, they use approximations to restrict severity of the problem. The insufficient accuracy induced by these approximations is not desirable. Simulated annealing (SA) can ameliorate this undesirable characteristic by simulating the physical

annealing process for the computation of the global or near-global optimum solutions for optimization problems. Amongst other applications, the SA technique has been successfully applied to economic dispatch [7] and hydrothermal scheduling [8].

As described in the synopsis above the EELD problem comprising the aforesaid objectives is converted into a single objective optimization problem using goal attainment (GA) method which is later on dealt by Simulated Annealing (SA) technique for seeking reasonably approximate global optimum solution in spite of presence of non-smooth unit characteristics. The algorithm developed has been implemented on 30-bus power system consisting of 5 thermal generators and 41 transmission lines. The experimental results are also presented.

## II. FORMULATION OF THE PROBLEM

The present formulation uses the Economic Emission Load Dispatch problem as a multi-objective optimization problem which is concerned with an attempt to optimize each objective simultaneously. Care is taken to see that the equality and inequality constraints of the system are satisfied. The following objectives and constraints are taken into consideration in the formulation of the Economic Emission Load Dispatch problem.

### A. Objectives

*Economy.* Consider a system having  $N$  buses and  $NL$  lines. Let the first  $NG$  buses have sources for power generation. Taking into account the valve-point effects[9], the fuel cost function of each generating unit is expressed as the sum of a quadratic and a squared sinusoidal function. Therefore, the total cost of generation  $C$  in terms of control variables  $PG$ 's is given by the following expression:

$$f_1(PG) = C = \sum_{i=1}^{NG} 0.5a_i PG_i^2 + b_i PG_i + c_i + |d_i \times \sin^2(ei \times (PG_i^{\min} - PG_i))| \$/h \quad (1)$$

where  $PG_i$  is the real power output of an  $i$ th generator,  $NG$

is the number of generators, and  $a_i, b_i, c_i, d_i, e_i$  is fuel cost curve coefficients of an  $i$ th generator.

**Emission.** The power generating stations being the primary sources of nitrous oxides, they are strongly objected by the Environmental Protection Agency to reduce their emissions. In this study, nitrous oxide ( $\text{NO}_x$ ) emission is

$$f_2(PG) = E = \sum_{i=1}^{NG} 0.5\alpha_i PG_i^2 + \beta_i PG_i + \gamma_i + \eta_i \exp(k_i PG_i) \quad lb/h \quad (2)$$

where  $\alpha_i, \beta_i, \gamma_i, \eta_i$  and  $k_i$  are emission curve coefficients of the  $i$ th generating unit.

**B. Line Security.**

Security constraints involve critical lines for replacing huge no of transmission lines in the power system network that are of immense importance in deciding the optimal solutions for an electric power systems. The system designer interprets the transmission lines violating the equality and inequality constraints as critical lines. The security constraints of the system can give better prospects by optimizing the following objective function:

$$f_3(PG) = S = \sum_{j=1}^k \left( |L_j(PG)| / L_j^{\max} \right) \quad (3)$$

where  $L_j(PG)$  is the real power flow,  $L_j^{\max}$  is the maximum limit of the real power flow of the  $j$ th line, and  $k$  is the number of monitored lines (critical lines). The line flow of the  $j$ th line is expressed in terms of the control variables  $PG$  's by utilizing the Generalized Generation Distribution Factors(GGDF) [11], and is given below:

$$F_j(PG) = \sum_{i=1}^{NG} (D_{ji} PG_i) \quad (4)$$

where  $D_{ji}$  is the generalized generation distribution factor (GGDF) for line  $j$  due to generator  $i$ .

**Load Constraint.** The real power balance between generation and the load is maintained always thinking the load at any time to be constant:

$$\sum_{i=1}^{NG} PG_i = P_D + P_L \quad (5)$$

where  $P_D$  is the total real power demand and  $P_L$  is the total real power loss. The latter is represented as [10]

$$P_L = \left[ \sum_{i=1}^{NG} (A_i PG_i) \right]^2 \quad (6)$$

where  $A_i$  is the loss coefficient due to the generator  $i$ .

The loss coefficient are evaluated from base load flow solution.

**Operating Constraints.** For achieving stable operation each generating unit is to be confined within its lower and upper real power limits.

$$PG_i^{\min} \leq PG_i \leq PG_i^{\max} \quad (7)$$

where  $PG_i^{\min}$  and  $PG_i^{\max}$  are the minimum and maximum real power output of  $i$ th unit, respectively.

taken as the selected index from the viewpoint of environment conservation. The amount of emission from each generator is given as a function of its output [8], which is the sum of a quadratic and an exponential function in the present work. Therefore, the total emission level  $E$  from all the units in the system can be expressed as

**III. THE GOAL-ATTAINMENT METHOD**

Multi-objective formulation index is dealt with a set of objectives  $f(x) = [f_1(x), f_2(x), \dots, f_n(x)]$ . In this method the designer sets a vector of designed goals  $g = [g_1, g_2, \dots, g_n]$  which form a powerful tool [13-15] that associate with aforesaid objectives. The level of attainment of the goals is controlled by a weight vector  $w = [w_1, w_2, \dots, w_n]$ .

In this GA method of optimization, the aforesaid nonlinear problem is solved:

$$\text{minimize } \lambda \quad x \in \Omega \text{ subject to } g + \lambda w \geq f(x), \quad \omega \in \Lambda_\epsilon \quad (8)$$

where  $x$  is a set of desired parameters which can be varied,  $\lambda$  is a scalar variable which introduces an element of slackness in to the system,  $\Omega$  is a feasible-solution region that satisfies all the parametric constraints, and

$$\Lambda_\epsilon = \{ \omega \in \mathfrak{R}^n \text{ St. } \omega_i \geq \epsilon, \sum_{i=1}^n \omega_i = 1 \text{ and } \epsilon \geq 0 \}$$

Figure 1 illustrates two-dimensional goal-attainment method. The multi-objective optimization is concerned with the generation and selection of non-inferior solution points [15] to characterize the objectives where an improvement in one objective requires a degradation in the others.

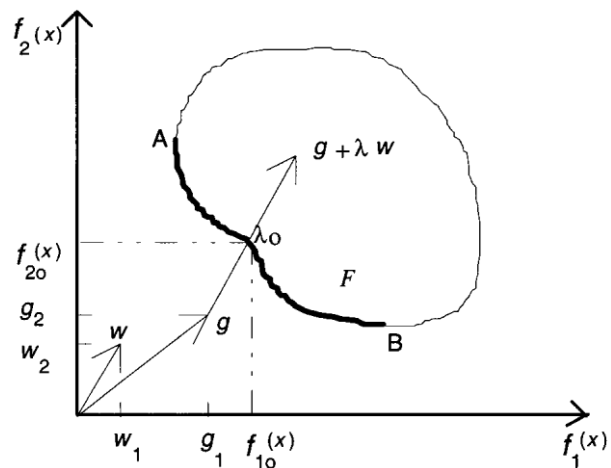


Figure 1: Illustration of two-dimensional goal-attainment method.

By varying  $w$  over  $\Lambda_\epsilon$ , the set of non-inferior solutions is generated. In the two dimensional representation of Figure 1, the set of non-inferior solutions lies on the curve AB. The weight vector  $w$  enables the designer to express a measure of the relative trade-offs between the objectives. Given the vectors  $w$  and  $g$ , the direction of the vector  $g + \lambda w$  is determined. A feasible

point on this vector in function space which is closest to the origin is then searched. The first point  $\lambda_0$  at which  $g + \lambda w$  intersects the feasible region  $F$  in the function space would be the optimal non-inferior solution. During the optimization,  $\lambda$  is varied, which changes the size of the feasible region. The constraint boundaries converge to the unique solution point  $\{f_{1o}(x), f_{2o}(x)\}$ . For optimizing the value of  $\lambda$  the Simulated Annealing method is employed as under:

**IV. SIMULATED ANNEALING TECHNIQUE**

In this method[16-17] a candidate solution is generated which is accepted when it becomes a better solution to generate another candidate solution. If it is deteriorated solution, the solution will be accepted provided its probability of acceptance  $P_r(\Delta)$  given by equation (9) is greater than an arbitrarily generated number between 0 and 1.

$$P_r(\Delta) = [1/\{1 + \exp(\Delta/T)\}] \tag{9}$$

where  $\Delta$  is the amount of deterioration between the new and the current solutions and  $T$  is the temperature at which the new solution is generated. In forming the new solution the current solution is perturbed [5] according to the Gaussian probability distribution function (GPDF). The mean of the GPDF is taken to be the current solution, and its standard deviation is given by the product of the temperature and a scaling factor  $\pm$ . The value of  $\pm$  is less than one, and together with the value of the temperature, it governs the size of the neighborhood space of the current solution and hence the amount of perturbation. The new solution is formed by adding the amount of perturbation to the current solution. In the next iteration the temperature is reduced according to a cooling schedule. The following geometric cooling schedule is adopted in the present work [15]:

$$T_v = r^{(v-1)} T_0 \tag{10}$$

where  $T_0$  and  $T_v$  are the initial temperature and the temperature at the  $v$ th iteration, respectively, and  $r$  is the temperature reduction factor. The solution process continues until the maximum number of iterations is reached and the optimum solution is found.

**V. EXPERIMENTAL RESULTS**

The algorithm developed in the previous section has been applied to a 30-bus test system. The system consists of 5 generators and 41 lines. The line data and the load data are given in the Appendix. Table 1 gives the real power operating limits whereas Tables 2 and 3 give the cost curve and emission curve coefficients of the 5 generators. The voltage at the 5 buses are kept fixed respectively, to the values 1.0634, 1.0482, 1.0354, 1.008 and 1.0631 p.u. .The system load was taken on 100MVA base.

In applying the developed algorithm for the test system, the appropriate values of the control parameters are set. These parameters are initial temperature  $T_0$ , the scaling factor  $\pm$  for GPDF, the temperature reduction factor  $r$ ,

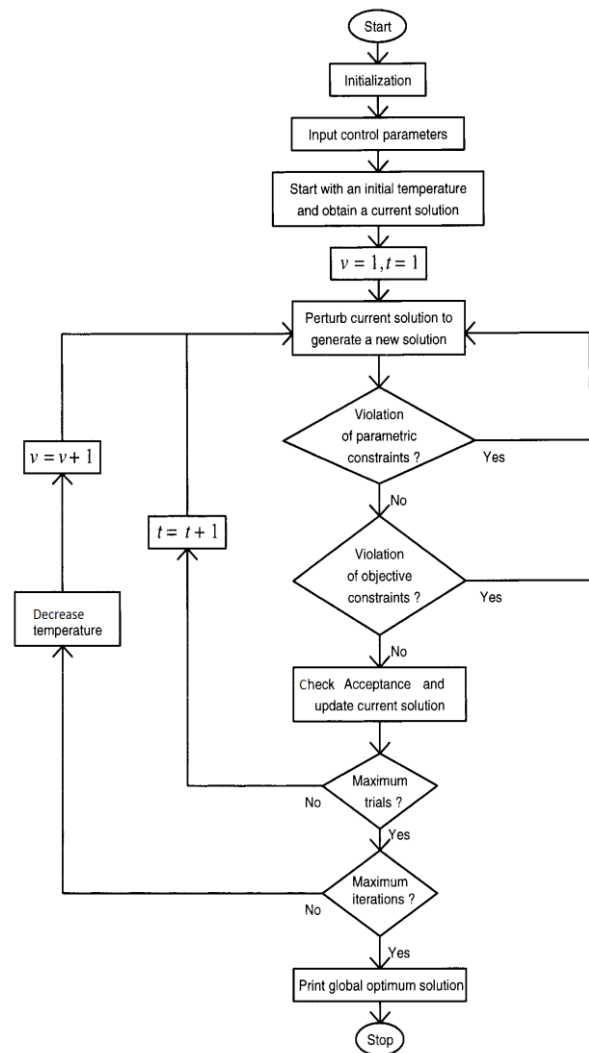


Figure 2: FLOW CHART FOR Multi-objective Economic Emission Load Dispatch with Nonlinear Fuel Cost and non-inferior Emission Level Functions

maximum number of iterations VMAX, and the number of trials per iteration TMAX. In the present work  $T_0$ ,  $\pm$ , VMAX, and TMAX were set, respectively, to the values of 50000, 0.02, 200, and 2000. As per a guideline [17], the value of  $r$  lies in the range from 0.80 to 0.99. SA with a slow cooling schedule usually has larger capacity to and the optimal solution than with a fast cooling schedule. Hence, for seeking the optimal solution the value of  $r$  is required to set close to 0.99 so that a slow cooling process is simulated. The appropriate setting of  $r$  was set by experimenting its value in the range from 0.95 to 0.99, and this value was found to be 0.98. The following different case studies were conducted to illustrate the performance of the proposed algorithm.

The variation of cost and emission functions with the optimized real power generation for the five test case systems is illustrated in plot-1 and plot-2 respectively.

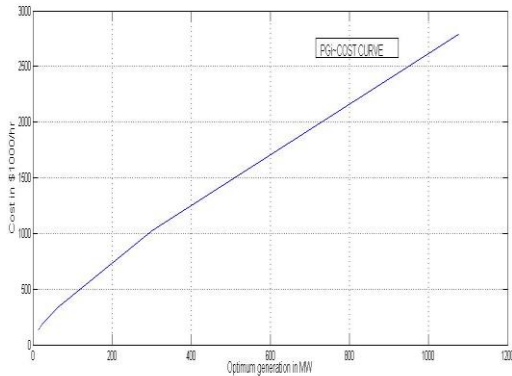


Figure 3: Variation of cost function  $f_1(PG_i)$  with  $PG_i$  (plot-1)

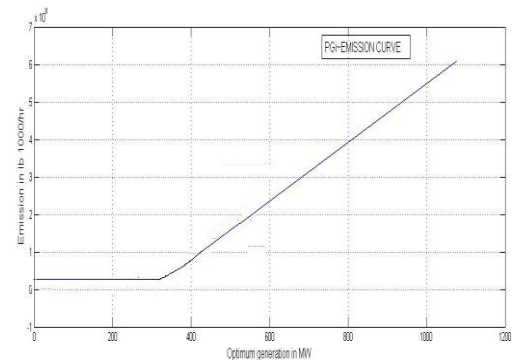


Figure 4: Variation of emission level function  $f_2(PG_i)$  with  $PG_i$  (plot-2)

The variation of cost and emission functions with the optimized real power generation for the seven test case systems is illustrated in plot-3 and plot-4 respectively.

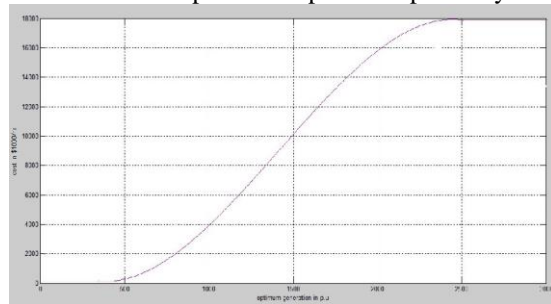


Figure 5: Variation of cost function  $f_1(PG_i)$  with  $PG_i$  (plot-3)-IEEE-57BUS SYSTEM

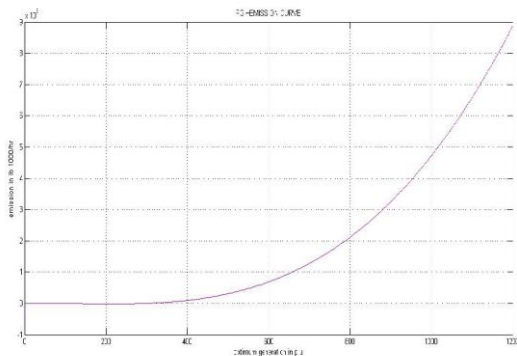


Figure 6: Variation of emission level function  $f_2(PG_i)$  with  $PG_i$  (plot-4)

This shows a better computation result as the number of buses get extended at the cost of little high time of computation. However the computation time can be reduced further by using advanced computing machine and differential evolutionary technique algorithm for multi-objective minimization.

**Table 1:** Operating limits (p.u.) of generators on 100 MVA base

Gen	$PG_{min_i}$	$PG_{max_i}$
1	0.5	3.00
2	0.2	1.25
3	0.3	1.75
4	0.1	4.75
5	0.4	11.50

**Table 2:** Cost curve coefficients of generators

	$a_i$	$b_i$	$c_i$	$d_i$	$e_i$
1	0.0015	1.8000	40.0	200	0.035
2	0.0030	1.8000	60.0	140	0.040
3	0.0012	2.1000	100.0	160	0.038
4	0.0080	2.0000	25.0	100	0.042
5	0.0010	2.0000	120.0	180	0.037

Emission curve coefficients of generators

**Table 3:** Cost curve coefficients of generators

Gen # $i$	$\alpha_i$	$\beta_i$	$\gamma_i$	$\eta_i$	$k_i$
1	0.0015	1.8000	40.0	200	0.035
2	0.0030	1.8000	60.0	140	0.040
3	0.0012	2.1000	100.0	160	0.038
4	0.0080	2.0000	25.0	100	0.042
5	0.0010	2.0000	120.0	180	0.037

### Case 1

A goal vector  $g = [g_1, g_2, g_3]^T = [357.1, 228.05, 1.1503]^T$  was generated automatically using Step 1(iv) of the computational algorithm. Here,  $g_1$  is the generating cost objective being expressed in \$/ h,  $g_2$  is the emission level objective being expressed in lb/ h, and  $g_3$  is the line security objective. The vector  $w = [w_1, w_2, w_3]^T = [0.3, 0.5, 0.5]^T$  that signifies the preference direction of the designer towards the goals was given by the designer.

### Case 2

A goal vector of  $g = [389, 251.06, 1.61]^T$  along with an identical weight vector as in case 1 is considered in this case. The goal values were assumed given by the Designer through his/ her experiences.

The optimum generation schedule was determined for both cases and are presented in Table 4.

The following three additional cases are also considered in the present study to observe the closeness of the value of a particular objective function towards its goal.

### Case 3

A goal vector of  $g = [399, 252, 1.90]'$  along with a weight vector of  $w = [0.4, 0.3, 0.5]'$  is considered.

#### Case 4

In this case, a goal vector of  $g = [400, 257, 2.100]'$  and a weight vector of  $w = [0.5, 0.4, 0.5]'$  are considered.

#### Case 5

This case considers a goal vector of  $g = [397, 227, 2.30]'$  and a weight vector of  $w = [0.5, 0.5, 0.4]'$ .

Case 3, case 4, and case 5 were computed using the proposed algorithm with same control parameters as in the previous two cases. The optimum generation schedule and the values of objective functions for these three cases are presented in Table 4.

The algorithm has been implemented in the above method using MATLAB programming language and the software system are run on a 2.53GHz computers.

## VI. CONCLUSION

The simulated annealing method along with goal attainment method was used to solve the aforesaid EELD problem with non-linear cost and emission functions characteristics in function space. Specifically the squared value of sin term attached to the cost function minimize the generation cost as depicted by the result analysis through table 1-5. An advantage of the proposed method is that it does not impose any convexity restrictions on the generating unit characteristics. In addition, it also allows the Designer to decide on different preferences for the objectives toward the goals according to the system operating conditions, thus resulting in a more flexible operation on generating units. The only demerit of the proposed method is longer execution time that can be improved by further developing the algorithms and using advance processors for computation purpose.

## VII. ACKNOWLEDGMENT

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